

Violation of Chandrasekhar Mass Limit: The Exciting Potential of Strongly Magnetized White Dwarfs

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Abstract

We consider a relativistic, degenerate, electron gas under the influence of a strong magnetic field, which describes magnetized white dwarfs. Landau quantization changes the density of states available to the electrons, thus modifying the underlying equation of state. In the presence of very strong magnetic fields a maximum of either one, two or three Landau level(s) is/are occupied. We obtain the mass-radius relations for such white dwarfs and their detailed investigation leads us to propose the existence of white dwarfs having a mass $\sim 2.3M_{\odot}$, which overwhelmingly exceeds the Chandrasekhar mass limit.

Introduction

Chandrasekhar first showed the maximum possible mass for a white dwarf (WD) to be $\sim 1.44M_\odot$ [1]. Later, several magnetized WDs have been discovered with surface fields of $10^5 - 10^9$ G [2], [3], [4], [5], [6]. Anticipating that the central field, which cannot be probed directly, will be much stronger than that on the surface, Ostriker and Hartwick [7] constructed models of WDs with $B \sim 10^{12}$ G at the center but with a much smaller field at the surface. Recent observations of peculiar Type Ia supernovae - SN2006gz, SN2007if, SN2009dc - seem to suggest super-Chandrasekhar mass WDs as their most likely progenitors [8]. These WDs, found in binary systems, stabilize by rotating and accreting matter [9]. In this essay we propose a mechanism by which the WDs exceed the Chandrasekhar mass limit in presence of strong magnetic fields at their centers. In a simple theoretical framework existence of such stars has been reported recently [10].

Here we consider strongly magnetized WDs having interior magnetic field $\gtrsim 10^{15}$ G. These Landau quantized electronic systems have at the most one, two or three occupied Landau level(s) (LL(s)). We obtain an exciting possibility of a mass of WDs $\sim 2.3M_\odot$.

Equation of state and solution procedure

Basic equations

The Landau quantized energy states of a free electron in a uniform, static, magnetic field are given by [11]

$$E_{\nu, p_z} = [p_z^2 c^2 + m_e^2 c^4 (1 + \nu \frac{2B}{B_c})]^{1/2}, \quad (1)$$

where ν denotes the LL, given by

$$\nu = j + \frac{1}{2} + \sigma, \quad (2)$$

when j being the principal quantum number, $\sigma = \pm \frac{1}{2}$, p_z the momentum of the electron along the z -axis, B the magnetic field, B_c a critical magnetic field defined by

$$B_c = m_e^2 c^3 / \hbar e = 4.414 \times 10^{13} \text{ G}, \quad (3)$$

where e is the charge of the electron, c the speed of light, m_e the rest-mass of the electron, \hbar the Planck's constant. Our interest is to study the effect of $B > B_c$ on the relativistic, degenerate electron gas.

The Fermi energy of electrons in units of $m_e c^2$ for a given ν is given by

$$\epsilon_F^2 = x_F(\nu)^2 + (1 + 2\nu \frac{B}{B_c}), \quad (4)$$

where $x_F(\nu)$ is the Fermi momentum in units $m_e c$.

As $x_F(\nu)^2 \geq 0$, the maximum number of occupied LLs

$$\nu_m = \left(\frac{\epsilon_{Fmax}^2 - 1}{2B_D} \right)_{\text{nearest lowest integer}}, \quad (5)$$

where $B_D = B/B_c$, ϵ_{Fmax} the dimensionless maximum Fermi energy of the system. Following [11] we write the electron number density

$$n_e = \frac{2B_D}{(2\pi)^2 \lambda_e^3} \sum_{\nu=0}^{\nu_m} g_\nu x_F(\nu), \quad (6)$$

where the Compton wavelength of the electron $\lambda_e = \hbar/m_e c$; the matter density

$$\rho = \mu_e m_H n_e, \quad (7)$$

where μ_e is the mean molecular weight per electron (which we choose to be 2) and m_H the mass of hydrogen atom; the electron degeneracy pressure

$$P = \frac{2B_D}{(2\pi)^2 \lambda_e^3} m_e c^2 \sum_{\nu=0}^{\nu_m} g_\nu (1 + 2\nu B_D) \eta \left(\frac{x_F(\nu)}{(1 + 2\nu B_D)^{1/2}} \right), \quad (8)$$

where

$$\eta(y) = \frac{1}{2} y \sqrt{1 + y^2} - \frac{1}{2} \ln(y + \sqrt{1 + y^2}). \quad (9)$$

Procedure

From equation (5) we see that if $B_D \gg 1$, then ν_m is small, implying electrons are restricted to the lower LLs. Since we are interested in this regime, depending on the specific values of B_D and ϵ_{Fmax} , we have one-level ($0 \leq \nu_m < 1$), two-level ($1 \leq \nu_m < 2$) and three-level ($2 \leq \nu_m < 3$) systems. By eliminating $x_F(\nu)$ numerically from equations (7) and (8), we obtain the $P - \rho$ relation which is the equation of state (EOS). Figure 1 shows EOSs for the different cases given in Table 1.

Table 1
Parameters for the equations of state in Figure 1.

ϵ_{Fmax}	ν_m	B_D	B in units of 10^{15} G
2	1	1.5	0.066
	2	0.75	0.033
	3	0.5	0.022
20	1	199.5	8.81
	2	99.75	4.40
	3	66.5	2.94
200	1	19999.5	882.78
	2	9999.75	441.38
	3	6666.5	294.26

The hydrostatic equilibrium of the WD in presence of constant magnetic field is described by [12]

$$\frac{1}{r^2} \frac{d}{dr} \left(\frac{r^2}{\rho} \frac{dP}{dr} \right) = -4\pi G \rho, \quad (10)$$

with the boundary conditions:

$$\rho(r = 0) = \rho_c \quad (11)$$

and

$$\left(\frac{d\rho}{dr}\right)_{r=0} = 0, \quad (12)$$

where ρ_c is the central density of the WD.

The radius R of the WD is obtained by solving equation (10) and is the value of r where density goes to zero. The mass M of the WD, which is approximated to be spherical, is obtained by integrating the following equation:

$$\frac{dM}{dr} = 4\pi r^2 \rho. \quad (13)$$

Constructing white dwarfs

Equations of State

Let us consider the case $\epsilon_{Fmax} = 20$. The solid curve in Figure 1(b) is free of any kink (only ground LL occupied). The dotted and dashed curves have one kink (ground and first LLs occupied) and two kinks (ground, first and second LLs occupied) respectively. Any kink represents a transition from a lower to upper LL.

Mass-radius relations

Table 2

Parameters for the fitting function for the equation of state shown in Figure 1(d).

ρ_D in units of $2 \times 10^9 \text{ gm/cc}$	Γ	K in CGS units
0 – 0.096	2.9	4.055
0.096 – 0.307	2.4	1.284
0.307 – 1.128	2.1	0.914
1.128 – 2.117	0.35	1.105
2.117 – 2.956	4/3	0.522
2.956 – 3.842	2.0	0.246
3.842 – 4.651	0.35	2.225
4.651 – 6.116	4/3	0.496
6.116 – 7.37	2.0	0.142

Figure 2 shows the mass-radius relations, where each point in the curves corresponds to a WD with a particular value of ρ_c which is supplied as a boundary condition.

In Figure 2(a) we see that as ρ_c increases, both M_D and R_D increase and then at higher values of ρ_c , R_D becomes nearly independent of M_D . We note that the most massive WD

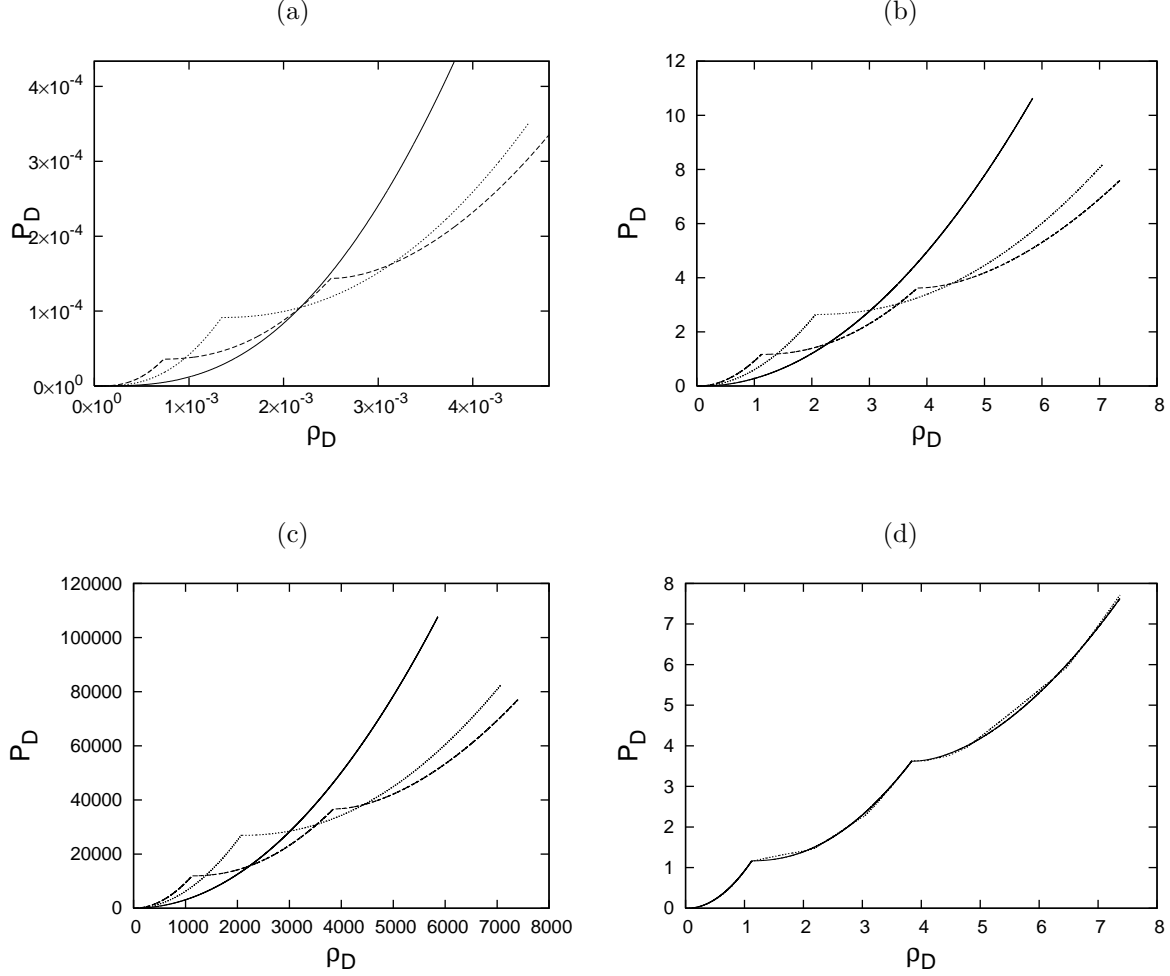


Figure 1: Equations of state in a strong magnetic field (given in Table 1) for (a) $\epsilon_{Fmax} = 2$, (b) $\epsilon_{Fmax} = 20$, (c) $\epsilon_{Fmax} = 200$. In all three cases the solid, dotted and dashed lines indicate one-level, two-level and three-level systems respectively. In (d) the solid line is same as the dashed line in (b), but fitted with the dotted line by analytical formalism (see text for details). Here P_D is the pressure in units of 2.668×10^{27} erg/cc and ρ_D is the density in units of 2×10^9 gm/cc.

on this curve has a mass $\sim 2.3 M_\odot$, which corresponds to the maximum density point of the solid curve in Figure 1(b). This denotes the density at which the ground LL is completely filled. Thus a WD with this ρ_c and a magnetic field strength of $B = 8.81 \times 10^{15}$ G has a mass greater than the Chandrasekhar limit (for details see [13]).

The turning point in Figure 2(b), after attaining the maximum mass $\sim 2.3 M_\odot$, corresponds to the kink in the corresponding EOS given in Figure 1(b). During the transition from ground to first LL, the radius and mass both decrease with increasing ρ_c . Then there is a brief range of densities where the mass increases as the radius decreases and ultimately at very high densities the radius is nearly independent of the mass.

In Figure 2(c) we see a repetition of the features in Figure 2(b) twice, which corresponds to the two kinks in the corresponding EOS in Figure 1(b). (The maximum mass $\sim 2.3 M_\odot$ occurs at that ρ_c where the ground LL is completely filled and transition to the first LL is about to start.)

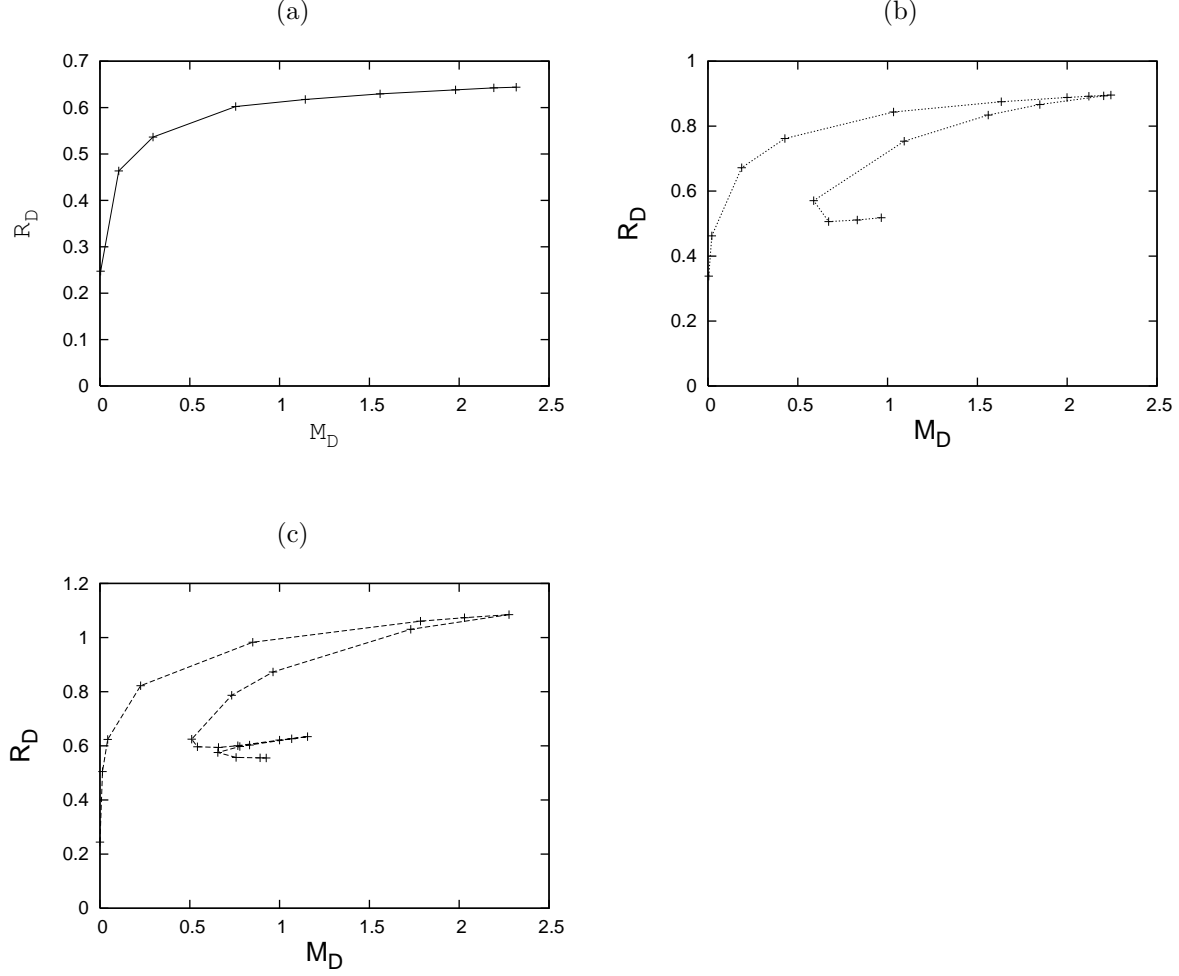


Figure 2: Mass-radius relations with $\epsilon_{Fmax} = 20$ for (a) one-level system, (b) two-level system, (c) three-level system. Here M_D is the mass of the white dwarf in units of M_\odot and R_D is its radius in units of 10^8 cm (the solid, dotted and dashed lines have the same meaning as in Figure 1).

For comparison let us recall the Lane-Emden solution for the classical non-magnetic case, which assumes a polytropic EOS

$$P = K\rho^\Gamma, \quad (14)$$

giving rise to the following relations [12]:

$$R \propto \rho_c^{\frac{\Gamma-2}{2}} \quad (15)$$

and

$$M \propto \rho_c^{\frac{3\Gamma-4}{2}}. \quad (16)$$

This means that if $\Gamma > 2$, R increases with ρ_c , if $\Gamma = 2$, R is independent of ρ_c , if $\Gamma > 4/3$, M increases with ρ_c and if $\Gamma = 4/3$, M is independent of ρ_c . This is exactly what is observed in Figure 2, if the EOSs given in Figure 1 are fitted by the polytropic EOS, adopting constant values of Γ in different density ranges (see Table 2). For instance in

Figure 1(b) (dashed line) and Figure 2(c), at very low densities, $\Gamma = 3$ and up to the turning point density (the kink in the EOS) the radius keeps increasing with mass and then becomes nearly independent of mass when $\Gamma \sim 2$. Then Γ suddenly drops to a small value ~ 0.35 which marks the onset of the transition from ground LL to first LL. In this region the pressure becomes independent of density, revealing an unstable zone in the EOS. As the density increases further, Γ approaches the relativistic value of $4/3$. In this regime we see that the radius decreases slightly as the mass does not change significantly. Next Γ takes up a value of 2 and the above phenomena will be repeated.

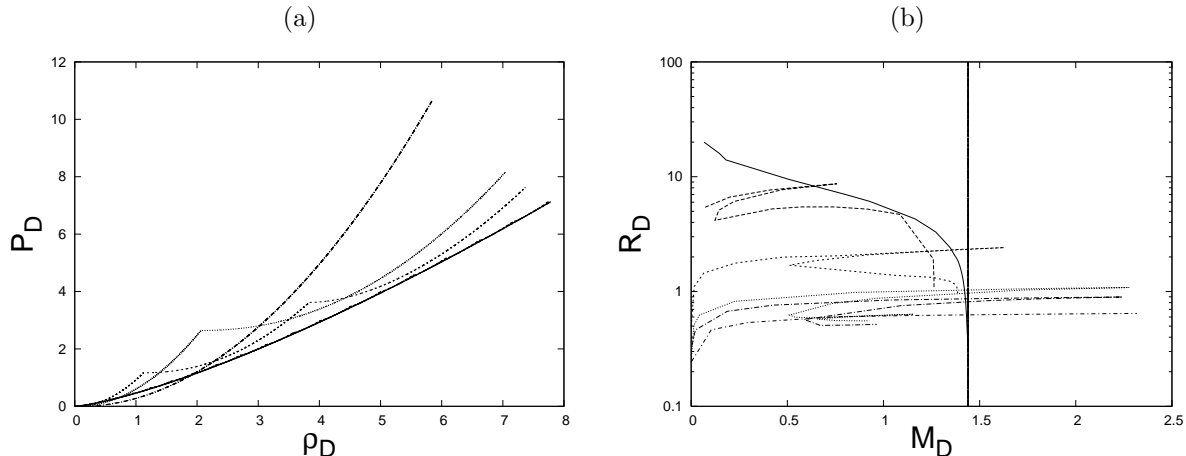


Figure 3: Comparison with Chandrasekhar's non-magnetic results for $\epsilon_{Fmax} = 20$. (a) Equations of state - the solid line represents Chandrasekhar's equation of state. The dot-dashed, dotted and dashed lines represent the one-level, two-level and three-level systems respectively. The equation of state for $\nu_m = 20$ is also shown, which appears as a series of kinks on top of the solid line. (b) Mass-radius relations - the vertical line marks the $1.44M_\odot$ limit and the solid line represents Chandrasekhar's mass-radius relation. From top to bottom the other lines represent $\nu_m = 500, 20, 3, 2$ and 1 respectively (the y -axis is in log scale).

Discussions

In Figure 3 we put together our results along with that of Chandrasekhar. Figure 3(a) clearly indicates that as the magnetic field decreases, the EOS approaches Chandrasekhar's EOS.

Figure 3(b) represents the corresponding mass-radius relations. We observe that as the magnetic field decreases, the mass-radius relation approaches that obtained by Chandrasekhar. We also note that as the magnetic field increases the WDs become more and more compact in size and the probability that they will have masses exceeding the Chandrasekhar limit also increases.

Figure 4 shows the variation of mass as a function of density within a magnetized WD for three different magnetic fields. In all the cases, we note that, by the time the density falls to about half the value of ρ_c , the mass has already crossed the Chandrasekhar limit as indicated by the horizontal line. Hence the effect of magnetic field is restricted to the high density regime, where the field remains essentially constant even in reality. This

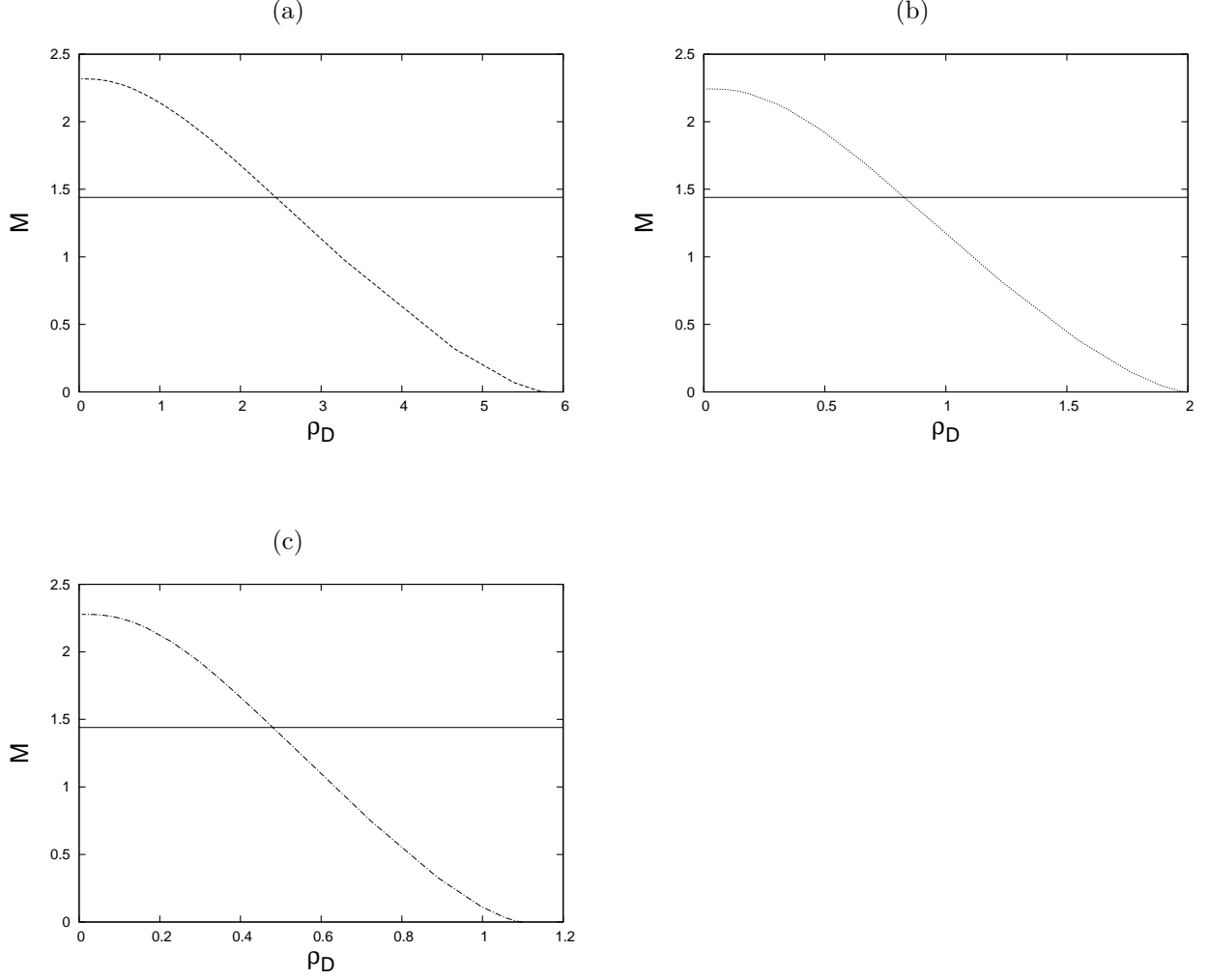


Figure 4: Mass as a function of density inside a magnetized white dwarf with B_D (a) 199.5, (b) 99.75, (c) 66.5, with $\epsilon_{Fmax} = 20$. The horizontal line indicates the $1.44M_\odot$ limit. See Table 1 for details.

justifies our choice of constant field.

Conclusions

We summarize the important findings of this work as follows:

- A transition from the lower to upper LL represents a kink in the EOS. If ρ_c lies at the kink appearing at the ground to first LL transition, the WD has the maximum possible mass.
- The most interesting result obtained is that there are possible WDs whose mass exceeds the Chandrasekhar limit and is found to be about $2.3M_\odot$. For instance, for $\epsilon_{Fmax} = 20$ and $B = 2.94 \times 10^{15}$ G, we obtain such a WD with $\rho_c = 2.2 \times 10^9$ gm/cc. Interestingly, the nature of mass-radius relations does not depend on the value of

ϵ_{Fmax} , however ϵ_{Fmax} determines how relativistic the system is. For instance, the Chandrasekhar limit is not exceeded for $\epsilon_{Fmax} = 2$, no matter what ρ_c is.

- As the magnetic field increases the WDs become more compact in size.
- The minimum magnetic field required to have a $2.3M_{\odot}$ WD is $B = 2.94 \times 10^{15}$ G. The magnetic field of the original star of radius R_{\odot} , which collapses into a WD of radius $\sim 10^8$ cm with the above field at its center, then turns out to be $\sim 6 \times 10^9$ G, based on the flux freezing theorem. Existence of such stars is not ruled out [14].
- In principle, the strong magnetic field causes the pressure to become anisotropic [15], [16]. As a result, the combined fluid-magnetic medium develops a magnetic tension [17], leading to a deformation in the WDs along the direction of the magnetic field. The WDs hence could adopt a flattened shape [13].
- The flattening effect due to magnetic field leads to super-Chandrasekhar WDs even for smaller magnetic fields. Such relatively weakly magnetized WDs are more probable in nature.
- We end by addressing the possible reason for not observing such a high field yet in a WD. This could be due to the magnetic screening effects on the surface of the WD if it is an accreting one. In this case the current in the accreting plasma depositing on the surface, presumably creates an induced magnetic moment of sign opposite to that of the original magnetic dipole, thus reducing the surface magnetic field of the WD, without affecting the central field. Hence by estimating the surface field alone one should not assume the rest.

Acknowledgments

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